2026 Team Tryout Problem Set

Los Gatos Math Circle

Due: Friday, August 28, 2026

Eight olympiad problems across the main contest topics. Work through as many as you can. You are **not** expected to solve all eight; a few complete, well-written solutions tell us far more than eight rushed ones. Write full solutions explaining your reasoning, work on your own, and submit a single PDF.

Problem 1. *(Algebra)*

Find all polynomials $P(x)$ with real coefficients such that

$$P(x)\,P(x+1) = P\big(x^2+x+1\big) \qquad \text{for all real } x.$$

Problem 2. *(Number theory)*

Prove that for every integer n, the number $n^5 - n$ is divisible by 30.

Problem 3. *(Combinatorics)*

Each cell of a 3×7 grid is colored black or white. Prove that there exist two rows and two columns such that the four cells lying at their intersections all have the same color.

Problem 4. *(Inequalities)*

Let a, b, c be positive real numbers. Prove that

$$\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} \ \geq \ \frac{3}{2}.$$

Problem 5. *(Geometry, Challenge)*

Let ABC be a triangle with incenter I. Line AI meets the circumcircle of ABC again at M. Prove that

$$MB = MC = MI.$$

Problem 6. *(Functional equations)*

Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that, for all real x and y,

$$f(x+y) = f(x) + f(y) \qquad \text{and} \qquad f(xy) = f(x)\,f(y).$$

Problem 7. *(Invariants)*

The numbers $1, 2, 3, \ldots, 2026$ are written on a board. In one move you erase two numbers a and b and write their absolute difference $|a-b|$. After 2025 moves a single number remains. Prove that this number is odd.

Problem 8. *(Number theory, Challenge)*

Find all integer solutions (x, y, z) of the equation

$$x^2 + y^2 + z^2 = 2xyz.$$